

**Economics 381, Fall 2016**

**Problem Set 1 – Mechanics of OLS**

**Due Date: August 30**

**Instructions:** Complete the problem set answers in the spaces given. You may find the study notes on linear algebra helpful. Do the calculations separately, clearly showing your work, and **staple** your calculations to this sheet.

**Name (printed):**

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Governor Haslam wants you to investigate the effect of education on wages. He speculates that the relationship between years of education ( $X$ ) and annual income ( $Y$ ) can be represented by a linear model such that for each individual  $i$ ,  $Y_i = \alpha + \beta X_i + \epsilon_i$  where  $\alpha$  is an intercept (i.e. expected wages if  $X_i = 0$ ),  $\beta$  is the increase in wages associated with one more year of education, and  $\epsilon_i$  is the error term.

To analyze this problem using Ordinary Least Squares (OLS) estimation, you gather survey information from two individuals on their income and education. The data you receive are below:

Income	Years of Education
$\begin{bmatrix} 39 \\ 49 \end{bmatrix}$	$\begin{bmatrix} 14 \\ 18 \end{bmatrix}$

Q1. What is the  $2 \times 1$  vector  $Y$ ?

$$Y = \begin{bmatrix} \phantom{0} \\ \phantom{0} \end{bmatrix}$$

Q2. What is the  $2 \times 2$  matrix  $X$ ? (Note: the first column, which is necessary to calculate the intercept  $\alpha$ , is already complete.)

$$X = \begin{bmatrix} 1 & \phantom{0} \\ 1 & \phantom{0} \end{bmatrix}$$

Q3. What is the  $2 \times 2$  matrix  $X'$  i.e. the transpose of  $X$ ?

$$X' = \begin{bmatrix} \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} \end{bmatrix}$$

Q4. Compute  $(X'X)$

$$(X'X) = \begin{bmatrix} \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} \end{bmatrix}$$

Q5. Perform a quick calculation to demonstrate that the determinant of  $(X'X)$  is 16.

Q6. Denote by  $C$  the adjoint of  $(X'X)$ . What is  $C$ ?

$$C = \begin{bmatrix} & \\ & \end{bmatrix}$$

Q7. Compute  $(X'X)^{-1}$ , i.e. the inverse of  $(X'X)$ . (Hint: the last two questions might be useful here.)

$$(X'X)^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Q8. Compute the  $2 \times 1$  vector  $(X'Y)$ .

$$(X'Y) = \begin{bmatrix} \\ \end{bmatrix}$$

Q9. Compute  $(X'X)^{-1}(X'Y)$

$$(X'X)^{-1}(X'Y) = \begin{bmatrix} \\ \end{bmatrix}$$

Q10. You have now calculated  $\alpha$  and  $\beta$  respectively for Governor Haslam. Complete the following sentence.

*“Using the available data, I conclude that increasing education by one year is associated with an additional \_\_\_\_\_ dollars in annual income.”*