Optimal Pigouvian Taxation when Externalities Affect Demand

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Abstract

Purchasing a network good such as a cell phone generates the positive externality of making phones more useful for others. Should we subsidize cell phones? If pollution/fumes from cars make commuting by foot less desirable, people become more willing to pay road tax. How then should we tax car pollution? To address these questions, I extend and generalize the standard optimal commodity tax model to let the demand for an externality-generating good depend on society's total consumption of that good. The optimal tax rate depends on three factors: the demand elasticities of the good, the marginal social cost, and the response of consumption to the externality. These factors are additive and separable. I find that if a negative externality affects consumption enough, the optimal policy can be to subsidize it.

1 Introduction

This paper derives the optimal tax rate for a commodity whose demand is affected by its own externality. Examples of such goods include automobiles, fashionable attire, credit card machines, languages, cell phones, and office software.

The contribution of this paper is a generalization of the result of Sandmo (1975), who in turn generalized Pigou (1920) and Ramsey (1927). When the government needs to raise revenue, how should we tax externality-generating goods? Sandmo showed the solution was an additive and separable weighted average of Pigouvian taxation and a form of the Ramsey inverse-elasticity rule. This paper extends that model, generalizing it to allow those goods whose demands are affected by externalities, and again finds that the solution has can be considered an additive and separable weighted average.

Many externalities do not directly affect demand. If a widget factory pollutes a lake, the demand for widgets does not change. However this general principle is inapplicable to many goods and services. The demand for cars is affected by traffic congestion. How fashionable a particular shoe is depends on how many others purchase the same style of shoe. There is little benefit to being the sole Esperanto-speaker in the city, or the only person with a fax machine. More generally, the demand for an important set of goods (including but not limited to network goods) depends on the consumption behavior of other individuals. To my knowledge, the optimal policy in relation these goods has not been investigated.

The taxation of these goods is thus of interest to public economists. However, this paper is not of solely theoretical interest. I argue that this model is applicable to a large class of goods. Society has observed the rise of fax machines, telephones, computer operating systems, and the internet. With technological progress, the relevance and prominence of network goods in particular will certainly significantly increase. Many will become part of the tax base. For this reason, this paper is relevant to policy as well as to public economic theory.

2 The Model

Consistent with the literature I model a government optimizing the unweighted sum of utilities for n identical consumers subject to a government budget requirement T. Lump sum taxes are infeasible, leisure is the untaxable good, and consumers purchase goods based on tax-inclusive prices. Each consumer chooses labor effort x_0, the wages of which act as a numeraire, and the complement of labor is leisure. The consumer's problem is thus to maximize

$$u^i = u(1 - x_0, x_1, \ldots, x_m, \alpha) \tag{1}$$

$$\mathscr{L} = u(.) + \lambda \left(x_0 - \sum_{i=1}^{m} P_i x_i \right) \tag{2}$$

In addition to labor, there are m taxable commodities in the economy. Commodity m generates an externality α, which for simplicity we will think of as total consumption of x_m. I denote u_i as the derivative of the utility function with respect to x_i, and therefore denote the derivative of utility with respect to α as u_{m+1}. If we are considering a negative externality, then $u_{m+1} < 0$. I assume that consumers do not consider their own effect on the externality, and that the usual conditions for an interior maximum hold.

The solution of the consumer's problem requires the following two first-order conditions:

$$FOC_1 : u_0 = \lambda \tag{3}$$

$$FOC_2 : u_i = \lambda P_i,\ i = 1, \ldots, m \tag{4}$$

Note also from the budget constraint that $-\frac{\partial x_0}{\partial P_k} + \sum_{i=1}^{m} P_i \frac{\partial x_0}{\partial P_k} + x_k = 0 \Rightarrow x_k = \frac{\partial x_0}{\partial P_k} - \sum_{i=1}^{m} P_i \frac{\partial x_i}{\partial P_k}$.

I follow convention by permitting governments to adjust a price vector P to maximize society's indirect utility $V(P)$:

$$V(P) = u\left[1 - x_0(P), x_1(P), \ldots, x_m\left(P, \alpha(P)\right), \alpha(P)\right)] \tag{5}$$

Note that this formulation permits both that demand for x_m be a function of α, and that α directly affects utility. The case of cars and air pollution may be a useful example. The welfare effect of adjusting the price of good k is:

$$\frac{\partial V(P)}{\partial P_k} = -u_0 \frac{\partial x_0}{\partial P_k} + \sum_{i=1}^{m-1} u_i \frac{\partial x_i}{\partial P_k} + u_m \left(\frac{\partial x_m}{\partial P_k} + \frac{\partial \alpha}{\partial P_k} \frac{\partial x_m}{\partial \alpha} \right) + u_{m+1} \frac{\partial \alpha}{\partial P_k} \tag{6}$$

$$= -u_0 \frac{\partial x_0}{\partial P_k} + \sum_{i=1}^{m} u_i \frac{\partial x_i}{\partial P_k} + u_m \left(\frac{\partial \alpha}{\partial P_k} \frac{\partial x_m}{\partial \alpha} \right) + u_{m+1} \frac{\partial \alpha}{\partial P_k} \tag{7}$$

Substituting in (3), (4):

$$= -\lambda \frac{\partial x_0}{\partial P_k} + \sum_{i=1}^{m} \lambda P_i \frac{\partial x_i}{\partial P_k} + u_m \left(\frac{\partial \alpha}{\partial P_k} \frac{\partial x_m}{\partial \alpha} \right) + u_{m+1} \frac{\partial \alpha}{\partial P_k} \tag{8}$$

$$= -\lambda \left(\frac{\partial x_0}{\partial P_k} - \sum_{i=1}^{m} P_i \frac{\partial x_i}{\partial P_k} \right) + u_m \left(\frac{\partial \alpha}{\partial P_k} \frac{\partial x_m}{\partial \alpha} \right) + u_{m+1} \frac{\partial \alpha}{\partial P_k} \tag{9}$$

And using the fact $x_k = \dfrac{\partial x_0}{\partial P_k} - \sum_{i=1}^{m} P_i \dfrac{\partial x_i}{\partial P_k}$, we conclude that

$$\frac{\partial V(P)}{\partial P_k} = -\lambda x_k + u_m \left(\frac{\partial \alpha}{\partial P_k} \frac{\partial x_m}{\partial \alpha} \right) + u_{m+1} \frac{\partial \alpha}{\partial P_k} \tag{10}$$

Equation (10) will be useful shortly. Define the government's problem as the maximization of $V(P)$ subject to raising a budget of at least T. Define t_i, the tax on good i, as the difference between the final price and the producer price: $t_i = P_i - p_i$. Implicitly this is assuming perfectly competitive production markets. This is an unreasonable assumption for many of the goods considered, but generalizing the optimal commodity tax to address for market imperfections is not the focus of this paper. The government maximization problem can be summarized as

$$\mathscr{L} = nV(P) - \beta \left[n \sum_{i=1}^{m} (P_i - p_i)x_i - T \right] \tag{11}$$

Now using Equation (10), we can see that a necessary condition for the optimal commodity tax rate is:

$$\frac{\partial \mathscr{L}}{\partial P_k} = -\lambda x_k + u_m \left(\frac{\partial \alpha}{\partial P_k} \frac{\partial x_m}{\partial \alpha} \right) + u_{m+1} \frac{\partial \alpha}{\partial P_k} - \beta \left[\sum_{i=1}^{m} t_i \frac{\partial x_i}{\partial P_k} + x_k \right] = 0 \tag{12}$$

This can be simplified. Noting that $\dfrac{\partial \alpha}{\partial P_k} = n \dfrac{\partial x_m}{\partial P_k}$,

$$\sum_{i=1}^{m} t_i \frac{\partial x_i}{\partial P_k} = -\left(\frac{\lambda + \beta}{\beta} \right) x_k + \frac{n}{\beta} \left(u_{m+1} + u_m \frac{\partial x_m}{\partial \alpha} \right) \frac{\partial x_m}{\partial P_k} \tag{13}$$

Using the terminology of the existing literature, let the coefficient matrix on t_i (the transpose of the Jacobian of the taxable goods' demand functions) be denoted $J^\star$. Further let $J \equiv det(J^\star)$ and denote J_{ik} as the cofactor of the element in row i, column j of J. Then, applying Cramer's Rule:

$$t_k = \left[-\left(\frac{\lambda + \beta}{\beta} \right) \right] \left(\frac{\sum_{i=1}^{m} x_i J_{ik}}{J} \right) + \frac{n}{\beta} \left(u_{m+1} + u_m \frac{\partial x_m}{\partial \alpha} \right) \left(\frac{\sum_{i=1}^{m} \frac{\partial x_m}{\partial P_i} J_{ik}}{J} \right) \tag{14}$$

As per Sandmo (1975), it can be shown that:

$$\sum_{i=1}^{m} \frac{\partial x_m}{\partial P_i} J_{ik} = \begin{cases} 0 & \text{for } k \neq m \\ J & \text{for } k = m \end{cases} \tag{15}$$

Consequently,

$$t_k = -\left(\frac{\lambda + \beta}{\beta} \right) \left(\frac{\sum_{i=1}^{m} x_i J_{ik}}{J} \right) + \frac{n}{\beta} \left(u_{m+1} + u_m \frac{\partial x_m}{\partial \alpha} \right) \times \mathbb{1}_{\{k=m\}} \tag{16}$$

$$\frac{t_k}{P_k} = \left(\frac{-1}{P_k} \right) \left(\frac{\lambda}{\beta} + 1 \right) \left(\frac{\sum_{i=1}^{m} x_i J_{ik}}{J} \right) + \frac{n}{\beta} \frac{\lambda}{\lambda} \frac{1}{P_k} \left(u_{m+1} + u_m \frac{\partial x_m}{\partial \alpha} \right) \times \mathbb{1}_{\{k=m\}} \tag{17}$$

Defining θ_i as the tax rate on good i, i.e. $\theta_i \equiv t_i/P_i$ and μ as the negative of the ratio of

Lagrangian multipliers, i.e. $\mu \equiv -\lambda/\beta$,

$$\theta_k = \left(\frac{-1}{P_k}\right)(1-\mu)\left(\frac{\sum_{i=1}^{m} x_i J_{ik}}{J}\right) - n\mu\left(\frac{1}{\lambda P_m}\right)\left(u_{m+1} + u_m \frac{\partial x_m}{\partial \alpha}\right) \times \mathbb{1}_{\{k=m\}} \tag{18}$$

Finally, substituting from (4) and rearranging, we have:

$$\theta_k = (1-\mu)\left[\frac{-1}{P_k}\frac{\sum_{i=1}^{m} x_i J_{ik}}{J}\right],\ k = 1, \ldots, (m-1) \tag{19}$$

$$\theta_m = (1-\mu)\left[\frac{-1}{P_m}\frac{\sum_{i=1}^{m} x_i J_{im}}{J}\right] - \mu\left[n\left(\frac{u_{m+1}}{u_m}\right)\right] - \mu\left[n\left(\frac{\partial x_m}{\partial \alpha}\right)\right] \tag{20}$$

This solves for the optimal tax rates. Equation (19) shows that the tax rate on the $m-1$ typical goods is a form of the Ramsey inverse elasticity rule, consistent with the findings of the previous literature.

Equation (20) defines the optimal rate for the m^{th} good. It shows that the tax comprises three additively separable components: the first, a Ramsey-like factor which decreases in the elasticity of the good; the second, a Pigouvian factor increasing in the magnitude of the direct externality; and the third, an adjustment for how consumption responds to the externality.

The departure of Equations (19) and (20) with previous research is the third 'consumption response' component. If consumption does not depend on the externality, e.g. when the demand for widgets in unaffected by pollution in a lake, the consumption response component is zero and the optimal tax rate collapses to that found by Sandmo (1975).

This can be shown more clearly by grouping the final terms in Equation (20) together:

$$\theta_m = (1-\mu)\left[\frac{-1}{P_m}\frac{\sum_{i=1}^{m} x_i J_{im}}{J}\right] + \mu\left[(-n)\left(\frac{u_{m+1}}{u_m} + \frac{\partial x_m}{\partial \alpha}\right)\right] \tag{21}$$

With this formulation, we can interpret the result as a weighted average (with weight μ) of Ramsey taxation and adjusted-externality taxation. There may be disutility caused by α, but the extent to which α increases consumption of x_m can mitigate that negative effect. Indeed, if $\frac{\partial x_m}{\partial \alpha} > \frac{\partial u_{m+1}}{\partial u_m}$, then the optimal policy is to effectively subsidize the negative externality.

2.1 Some Illustrative Examples

The optimal tax rate formula derived in this paper can be applied to a variety of contexts. In this section I illustrate the components of the model with three examples: vehicles and air pollution, vehicles and traffic congestion, and cell phones. Although this model is static, I also show how the intuition could be interpreted in an intertemporal setting.

Vehicles and Air Pollution

The emission of particulate matter and nitrogen oxides by cars and trucks are textbook examples of negative externalities. Let us assume that the direct effect of increased air pollution is to lower society's welfare, i.e. $u_{m+1} < 0$. Indirectly, perhaps beyond a certain threshold, air pollution would discourage commuting by foot. This would make car usage more appealing. Stated in the language of the model, $\frac{\partial x_m}{\partial \alpha} > 0$. Applying this logic to Equation (20), we note that the negative sign on u_{m+1} tends to increase the tax on cars, but that is counterbalanced by the positive sign on $\frac{\partial x_m}{\partial \alpha}$. Whether the tax rate θ_m is larger or smaller than the standard Ramsey rate depends on whether $u_{m+1} > \frac{\partial x_m}{\partial \alpha}$ or $u_{m+1} < \frac{\partial x_m}{\partial \alpha}$. In the latter case, the optimal tax rate is lower than the Ramsey rate: the optimal policy is to lower taxes (i.e. effectively subsidize) the negative externality.

Vehicles and Traffic Congestion

Traffic congestion, like air pollution, negatively affects social welfare. However it diverges from air pollution in that congested roads makes car ownership *less* desirable. Therefore in the language of the model, $u_{m+1} < 0$ and $\frac{\partial x_m}{\partial \alpha} < 0$. Here both components of the externality are undesirable and the tax rate unambiguously rises above the Ramsey rate.

Cell Phones

For this illustration, we assume that the use of cell phones generates no direct externalities.[1] That is, one person's cell phone use does not directly affect another's utility. However the act of joining a cell phone network has the effect of making cell phone usage more desirable for others. In this case, $u_{m+1} = 0$ and $\frac{\partial x_m}{\partial \alpha} > 0$. From Equation (20) the optimal policy is to treat the $\frac{\partial x_m}{\partial \alpha}$ component identically to a typical positive externality, and to lower the tax rate accordingly.

Dynamic Effects

The model of this paper is static. Just as the logic of homogenous agent models can be extended to models of heterogenous agents, discussion of dynamics is not precluded by the fact that this model is static. If the intuition of the model can be extended to a multi-period setting then we can make inferences about the intertemporal implications of the model. This allows us to consider the entire dynamic path of taxation, rather than simply the rate at any time t.

[1] It has been suggested that there are negative epidemiological consequences of increased electromagnetic radiation, but let us omit that for the sake of argument.

Suppose a network good is developed, that initially the good has only a few users, and thus demand that is quite elastic. It is plausible in this case that the principal/largest effect in the optimal tax formula is the $\frac{\partial x_m}{\partial \alpha}$ term. Then, by Equation (20), the good should initially be subsidized. If after a few years' growth the $\frac{\partial x_m}{\partial \alpha}$ tends to zero, the principal effect may well be the Ramsey taxation component. Under these conditions, the good should then be taxed. This intuition can clarify the counterintuitive logic that governments should subsidize early adoption of cell phones, and tax them when they become popular enough.

3 Conclusion

The primary contribution of this paper is the derivation of optimal tax rate for network goods. I developed a model where the demand for an externality-generating good is affected by its own externality. This setting is applicable quite broadly: to modern network goods like phones and operating systems, but also to a variety of other important non-network products like automobiles and clothing. Indeed any component of the utility function which is affected the society's total level of consumption can be investigated with this model. The optimal tax rate generalizes previous results from the literature, including those of Pigou (1920), Ramsey (1927), and Sandmo (1975). The tax rate comprises three additively separable factors related to substitution elasticities, the magnitude of the direct externality, and the effect of the elasticity on consumption behavior. Extending the model to heterogenous consumers is left for future work.

References

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